$$\Rightarrow \int_0^c h(x)\,p(x)\,dx - t\int_0^c h(x)\,f(x)\,dx \le$$

$$\le t\int_0^c h(x)\,f(x)\,dx - \int_0^c h(x)\,g(x)\,dx \Rightarrow$$

$$\Rightarrow \int_0^1 h(x)\,g(x)\,dx \le$$

$$\le \frac{\int_0^1 g(x)\,dx}{\int_0^1 f(x)\,dx}\int_0^1 h(x)\,f(x)\,dx \Rightarrow \int_0^1 h(x)\,g(x)\,dx \cdot \int_0^1 f(x)\,dx \le$$

$$\le \int_0^1 g(x)\,dx \cdot \int_0^1 h(x)\,f(x)\,dx$$

W9. (Solution by the proposer.) Let $S_k = F_2 + F_4 + \ldots + F_{2k}$. We claim that $S_k = F_{2k+1} - 1$. Indeed, we have

$$\begin{aligned} F_1 &= F_2 - F_0 \\ F_3 &= F_4 - F_2 \\ &\vdots \\ F_{2k-3} &= F_{2k-2} - F_{2k-4} \\ F_{2k-1} &= F_{2k} - F_{2k-2} \end{aligned}$$

Adding up the preceding expressions, yields

$$\sum_{i=1}^{k} F_{2i-1} = F_{2k} - F_0 .$$

Then,

$$\sum_{i=1}^{k} F_{2i} = \sum_{i=1}^{2k} F_i - \sum_{i=1}^{k} F_{2i-1} = (F_{2k+2} - 1) - F_{2k} = F_{2k+1} - 1$$

on account that

$$\sum_{i=1}^{2k} F_i = F_{2k+2} - 1$$

as can be easily proven by induction. Finally,

$$\sum_{k=1}^{n} \frac{F_{2k}}{(F_{2k+1}-1)^2} = \sum_{k=1}^{n} \frac{F_{2k}}{S_k^2} = 1 + \sum_{k=2}^{n} \frac{S_k - S_{k-1}}{S_k^2}$$

$$\leq 1 + \sum_{k=2}^{n} \left(\frac{1}{S_{k-1}} - \frac{1}{S_k} \right) = 1 + \frac{1}{S_1} - \frac{1}{S_n} < 1 + \frac{1}{F_2} = 2,$$

and we are done.

Second solution. Since $F_{2k-1} < F_{2k+1}, k \in \mathbb{N}$ then

$$\frac{F_{2k}}{(F_{2k+1}-1)^2} < \frac{F_{2k}}{(F_{2k-1}-1)(F_{2k+1}-1)} =$$

$$= \frac{F_{2k+1} - F_{2k-1}}{(F_{2k-1}-1)(F_{2k+1}-1)} = \frac{F_{2k+1} - 1 - (F_{2k-1} - 1)}{(F_{2k-1}-1)(F_{2k+1}-1)} =$$

$$= \frac{1}{F_{2k+1}-1} - \frac{1}{F_{2k+1}-1}$$

and, therefore

$$\sum_{k=1}^{n} \frac{F_{2k}}{(F_{2k+1}-1)^2} = \frac{F_2}{(F_3-1)^2} + \sum_{k=2}^{n} \frac{F_{2k}}{(F_{2k+1}-1)^2} = 1 + \sum_{k=2}^{n} \frac{F_{2k}}{(F_{2k+1}-1)^2} <$$

$$< 1 + \sum_{k=2}^{n} \left(\frac{1}{F_{2k+1}-1} - \frac{1}{F_{2k+1}-1} \right) = 1 + \left(\frac{1}{F_3 - 1} - \frac{1}{F_{2n+1}-1} \right) =$$

$$= 1 + \left(\frac{1}{2-1} - \frac{1}{F_{2n+1}-1} \right) < 2.$$

Arkady Alt